

Neural-Symbolic Recursive Machine for Systematic Generalization

Qing Li, Yixin Zhu, Yitao Liang, Ying Nian Wu, Song-Chun Zhu, Siyuan Huang



Introduction

Motivation: how can we build a model with human-like systematic generalization?

Hypothesis: a model with human-like systematic generalization is compositional and *equivariant*.

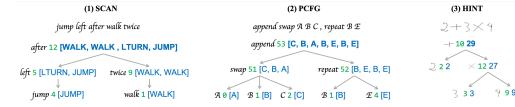
walk right \rightarrow RTURN WALK jump \rightarrow JUMP compositional $\Phi(T_c(x_1, x_2)) = T'_c(\Phi(x_1), \Phi(x_2))$ **walk** right after jump \longrightarrow JUMP RTURN **WALK** $\Phi(T_p x) = T'_p \Phi(x)$ equivariant **run** right after jump \longrightarrow JUMP RTURN **RUN**

Our contributions:

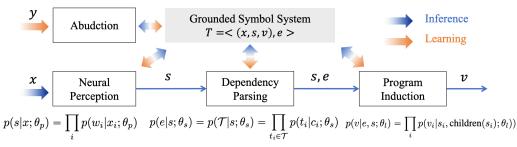
- 1. We propose Neural-Symbolic Recursive Machine (NSR), a principled framework for jointly learning perception, syntax, and semantics.
- 2. The core of NSR is a Grounded Symbol System (GSS), allowing for the emergence of combinatorial syntax and semantics directly from training data.
- 3. NSR achieves impressive systematic generalization across SCAN, PCFG, and HINT.

Code: https://liqing-ustc.github.io/NSR

Grounded Symbol System (GSS)



Neural-Symbolic Recursive Model (NSR)



Learning by Deduction-Abduction $7 + 4 \times 4 \times 23$

44

44

| | $7 + 4 \times 4 = 23$ | | | | |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|--|--|
| Algorithm A1: Learning by Deduction-Abduction | (1) abduce perception: change one symbol | | | | |
| Input :Training set $D = (x_i, y_i) : i = 1, 2,, N$ | $++35 \rightarrow ++23$ | | | | |
| Output : $	heta_p^{(T)}, 	heta_s^{(T)}, 	heta_l^{(T)}$ | $\overrightarrow{}$ $\phantom{$ | | | | |
| Initial Module : perception $\theta_p^{(0)}$, syntax $\theta_s^{(0)}$, semantics $\theta_l^{(0)}$ | | | | | |
| for $t \leftarrow 0$ to T do | 477 444 444 44 | | | | |
| Buffer $\mathcal{B} \leftarrow \emptyset$ | | | | | |
| foreach $(x, y) \in D$ do | (2) abduce syntax: rotate one arc | | | | |
| $T \leftarrow DEDUCE(x, \theta_p^{(t)}, \theta_s^{(t)}, \theta_l^{(t)})$ | $\times 44 \longrightarrow + 23$ | | | | |
| $T^* \leftarrow ABDUCE(T, y)$ | ++11 4 4 4 7 7 X × 16 | | | | |
| $\mathcal{B} \leftarrow \mathcal{B} \cup T^*$ | \sim | | | | |
| $\begin{bmatrix} \boldsymbol{\theta}_p^{(t+1)}, \boldsymbol{\theta}_s^{(t+1)}, \boldsymbol{\theta}_l^{(t+1)} \leftarrow \operatorname{learn}(\mathcal{B}, \boldsymbol{\theta}_p^{(t)}, \boldsymbol{\theta}_s^{(t)}, \boldsymbol{\theta}_l^{(t)}) \end{bmatrix}$ | 77 944 944 44 | | | | |
| return $	heta_p^{(T)}, 	heta_s^{(T)}, 	heta_l^{(T)}$ | (3) abduce semantics: change one <i>value</i> | | | | |
| | ++15 $++23$ | | | | |
| Function DEDUCE ($x,	heta_p,	heta_s,	heta_l$): | ₹ 77 X×8 ₹ 77 × 16 | | | | |
| Sample $\hat{s} \sim p(s x;\theta_p), \hat{e} \sim p(e \hat{s};\theta_s), \hat{v} = f(\hat{s},\hat{e};\theta_l)$ | | | | | |
| return $T = \langle (x, \hat{s}, \hat{v}), \hat{e} \rangle$ | 44 444 444 44 | | | | |

Experimental Results

NSR achieves perfect generalization in SCAN and PCFG.

| models | | | SCAN | PCFG | | | | | | |
|---------------------------------------------------------------------------------------------------------------|-------------------------|-----------------------|-------|-------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|--------------|--|--|
| mouchs | SIMPLE JUMP AROUND RIGH | | | | LENGTH | i.i.d. | systematicity | productivity | | |
| Seq2Seq | 99.7 | 1.2 | 2.5 | | 13.8 | 79 | 53 | 30 | | |
| CNN Î | 100.0 | 69.2 | 56.7 | | 0.0 | 85 | 56 | 31 | | |
| Transformer | - | - | - | | 20.0 | - | 96 | 85 | | |
| NeSS | 100.0 | 100.0 | 100.0 | | 100.0 | ≈ 0 | ≈ 0 | ≈ 0 | | |
| NSR (ours) | 100.0 | 100.0 | 100.0 | | 100.0 | 100 | 100 | 100 | | |
| turn walk look run jump left right opposite around twice thrice and after | 1000 | (67 19/1, 2000, | | - 0.7 - 0.5 - 0.2 | 0 walk : Ø look : Ø - jump : Ø - jump : Ø 0 left : x → ríght : x 5 opposíte around : 0 twíce : x thríce : x and : x, y | $\begin{aligned} urn: \emptyset \to [] & 1 \to WALK \\ alk: \emptyset \to [inc 0] & 2 \to LOOK \\ sok: \emptyset \to [inc inc 0] & 3 \to RUN \\ un: \emptyset \to [inc inc nc 0] & 5 \to LTRUN \\ un: \emptyset \to [inc inc inc inc nc 0] & 5 \to LTRUN \\ imp: \emptyset \to [inc inc inc inc inc inc 0, x) \\ ight: x \to cons (inc inc inc inc inc 0, x) \\ ight: x \to cons (inc inc inc inc inc 0, x) \\ round: x \to +(+(+(x, x), x), x) \\ wice: x \to +(+(x, x), x) \\ nice: x \to +(x, x) \\ nice: x, y \to +(x, y) \\ fter: x, y \to +(y, x) \\ (b) Programs induced using NSR. \end{aligned}$ | | | | |
| | - | input words in 1 | (b |) Progra | ms induced using | NSR. | | | | |

NSR outperforms Transformer by 23% in HINT.

| Model | Symbol Input | | | | | Image Input | | | | | | |
|-------------|--------------|-------------|-------------|-------------|-------------|-------------|------|------|------|------|------|------|
| | I | SS | LS | SL | LL | Avg. | I | SS | LS | SL | LL | Avg. |
| GRU | 76.2 | 69.5 | 42.8 | 10.5 | 15.1 | 42.5 | 66.7 | 58.7 | 33.1 | 9.4 | 12.8 | 35.9 |
| LSTM | 92.9 | 90.9 | 74.9 | 12.1 | 24.3 | 58.9 | 83.9 | 79.7 | 62.0 | 11.2 | 21.0 | 51.5 |
| Transformer | 98.0 | 96.8 | 78.2 | 11.7 | 22.4 | 61.5 | 88.4 | 86.0 | 62.5 | 10.9 | 19.0 | 53.1 |
| NeSS | ≈ 0 | ≈ 0 | ≈ 0 | ≈ 0 | ≈ 0 | ≈ 0 | - | - | - | - | - | - |
| NSR (ours) | 98.0 | 97.3 | 83.7 | 95.9 | 77.6 | 90.1 | 88.5 | 86.2 | 67.1 | 83.2 | 58.2 | 76.0 |

The evolution of learned programs is human-like.

| | master counting | master + and - | master × and ÷ # Training epochs |
|---------|------------------|--------------------------------------------------------|-------------------------------------------------|
| 0: Null | 0: 0 | 0:0 | 0:0 |
| 1: Null | 1: inc 0 | 1: inc 0 | 1: inc 0 |
| 2: Null | 2: inc inc 0 | 2: inc inc 0 | 2: inc inc 0 |
| | | | |
| 9: Null | 9: inc inc inc 0 | 9: inc inc inc 0 | 9: inc inc inc 0 |
| +: Null | +: Null | +: if $(y == 0, x, +(inc x, dec y))$ | +: if $(y == 0, x, (inc x) + (dec y))$ |
| -: Null | -: Null | -: if $(y == 0, x, +(dec x, dec y))$ | -: if $(y == 0, x, (dec x) + (dec y))$ |
| ×: Null | ×: Null | $\times: \text{ if } (y == 0, y, x)$ | $x: if (x == 0, 0, y \times (dec x) + y)$ |
| ÷: Null | ÷: Null | \div : if (y == inc 0, x, if (x == 0, x, inc inc 0)) | \div : if (x == 0, 0, inc ((x - y) \div y)) |
| | | | |